

Solving Equations

Main Ideas

- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.

New Vocabulary

open sentence
equation
solution

▶ GET READY for the Lesson

An important statistic for pitchers is the earned run average (ERA). To find the ERA, divide the number of earned runs allowed R by the number of innings pitched I . Then multiply the quotient by 9.

$$\begin{aligned}\text{ERA} &= \frac{R \text{ runs}}{I \text{ innings}} \times \frac{9 \text{ innings}}{1 \text{ game}} \\ &= \frac{9R}{I} \text{ runs per game}\end{aligned}$$



Verbal Expressions to Algebraic Expressions Verbal expressions can be translated into algebraic or mathematical expressions. Any letter can be used as a variable to represent a number that is not known.

EXAMPLE Verbal to Algebraic Expression

1 Write an algebraic expression to represent each verbal expression.

- three times the square of a number $3x^2$
- twice the sum of a number and 5 $2(y + 5)$

CHECK Your Progress

- the cube of a number increased by 4 times the same number
- three times the difference of a number and 8

A mathematical sentence containing one or more variables is called an **open sentence**. A mathematical sentence stating that two mathematical expressions are equal is called an **equation**.

EXAMPLE Algebraic to Verbal Sentence

2 Write a verbal sentence to represent each equation.

- $n + (-8) = -9$ The sum of a number and -8 is -9 .
- $\frac{n}{6} = n^2$ A number divided by 6 is equal to that number squared.

CHECK Your Progress

- $g - 5 = -2$
- $2c = c^2 - 4$

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a **solution** of the open sentence.

Properties of Equality To solve equations, we can use properties of equality. Some of these properties are listed below.



Vocabulary Link

Symmetric

Everyday Use having two identical sides

Math Use The two sides of an equation are equal, so the sides can be switched.

KEY CONCEPT

Properties of Equality

Property	Symbols	Examples
Reflexive	For any real number a , $a = a$.	$-7 + n = -7 + n$
Symmetric	For all real numbers a and b , if $a = b$, then $b = a$.	If $3 = 5x - 6$, then $5x - 6 = 3$.
Transitive	For all real numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.	If $2x + 1 = 7$ and $7 = 5x - 8$, then $2x + 1 = 5x - 8$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a .	If $(4 + 5)m = 18$, then $9m = 18$.

EXAMPLE Identify Properties of Equality

1 Name the property illustrated by each statement.

a. If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.

Transitive Property of Equality

b. If $12m = 24$, then $(2 \cdot 6)m = 24$.

Substitution



CHECK Your Progress

3. If $-11a + 2 = -3a$, then $-3a = -11a + 2$.

Sometimes an equation can be solved by adding the same number to each side, or by subtracting the same number from each side, or by multiplying or dividing each side by the same number.

KEY CONCEPT

Properties of Equality

Addition and Subtraction

Symbols For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.

Examples If $x - 4 = 5$, then $x - 4 + 4 = 5 + 4$.
If $n + 3 = -11$, then $n + 3 - 3 = -11 - 3$.

Multiplication and Division

Symbols For any real numbers a , b , and c , if $a = b$, then $a \cdot c = b \cdot c$, and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Examples If $\frac{m}{4} = 6$, then $4 \cdot \frac{m}{4} = 4 \cdot 6$. If $-3y = 6$, then $\frac{-3y}{-3} = \frac{6}{-3}$.



EXAMPLE Solve One-Step Equations**4** Solve each equation. Check your solution.

a. $a + 4.39 = 76$

$$a + 4.39 = 76 \quad \text{Original equation}$$

$$a + 4.39 - 4.39 = 76 - 4.39 \quad \text{Subtract 4.39 from each side.}$$

$$a = 71.61 \quad \text{Simplify.}$$

The solution is 71.61.

CHECK $a + 4.39 = 76 \quad \text{Original equation}$

$$71.61 + 4.39 \stackrel{?}{=} 76 \quad \text{Substitute 71.61 for } a.$$

$$76 = 76 \quad \checkmark \quad \text{Simplify.}$$

b. $-\frac{3}{5}d = 18$

$$-\frac{3}{5}d = 18 \quad \text{Original equation}$$

$$-\frac{5}{3}\left(-\frac{3}{5}\right)d = -\frac{5}{3}(18) \quad \text{Multiply each side by } -\frac{5}{3}, \text{ the multiplicative inverse of } -\frac{3}{5}.$$

$$d = -30 \quad \text{Simplify.}$$

The solution is -30 .

CHECK $-\frac{3}{5}d = 18 \quad \text{Original equation}$

$$-\frac{3}{5}(-30) \stackrel{?}{=} 18 \quad \text{Substitute } -30 \text{ for } d.$$

$$18 = 18 \quad \checkmark \quad \text{Simplify.}$$

CHECK Your Progress

4A. $x - 14.29 = 25$

4B. $\frac{2}{3}y = -18$

Study Tip**Multiplication and Division Properties of Equality**

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by $-\frac{3}{5}$ is the same as multiplying each side by $-\frac{5}{3}$.

EXAMPLE Solve a Multi-Step Equation**5** Solve $2(2x + 3) - 3(4x - 5) = 22$.

$$2(2x + 3) - 3(4x - 5) = 22 \quad \text{Original equation}$$

$$4x + 6 - 12x + 15 = 22 \quad \text{Apply the Distributive Property.}$$

$$-8x + 21 = 22 \quad \text{Simplify the left side.}$$

$$-8x = 1 \quad \text{Subtract 21 from each side to isolate the variable.}$$

$$x = -\frac{1}{8} \quad \text{Divide each side by } -8.$$

The solution is $-\frac{1}{8}$.**CHECK Your Progress**

Solve each equation.

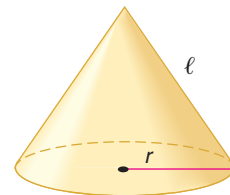
5A. $-10x + 3(4x - 2) = 6$

5B. $2(2x - 1) - 4(3x + 1) = 2$

You can use properties to solve an equation or formula for a variable.

EXAMPLE Solve for a Variable

- 6 GEOMETRY** The formula for the surface area S of a cone is $S = \pi r\ell + \pi r^2$, where ℓ is the slant height of the cone and r is the radius of the base. Solve the formula for ℓ .



$$S = \pi r\ell + \pi r^2 \quad \text{Surface area formula}$$

$$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2 \quad \text{Subtract } \pi r^2 \text{ from each side.}$$

$$S - \pi r^2 = \pi r\ell \quad \text{Simplify.}$$

$$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r} \quad \text{Divide each side by } \pi r.$$

$$\frac{S - \pi r^2}{\pi r} = \ell \quad \text{Simplify.}$$

CHECK Your Progress

- 6.** The formula for the surface area S of a cylinder is $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the base, and h is the height of the cylinder. Solve the formula for h .

STANDARDIZED TEST EXAMPLE

Apply Properties of Equality

- 7** If $3n - 8 = \frac{9}{5}$, what is the value of $3n - 3$?

A $\frac{34}{5}$

B $\frac{49}{15}$

C $-\frac{16}{5}$

D $-\frac{27}{5}$

Test-Taking Tip

Using Properties

If a problem seems to require lengthy calculations, look for a shortcut. There may be a quicker way to solve it. Try using properties of equality.

Read the Test Item

You are asked to find the value of $3n - 3$. Your first thought might be to find the value of n and then evaluate the expression using this value. Notice that you are *not* required to find the value of n . Instead, you can use the Addition Property of Equality.

Solve the Test Item

$$3n - 8 = \frac{9}{5} \quad \text{Original equation}$$

$$3n - 8 + 5 = \frac{9}{5} + 5 \quad \text{Add 5 to each side.}$$

$$3n - 3 = \frac{34}{5} \quad \frac{9}{5} + 5 = \frac{9}{5} + \frac{25}{5} \text{ or } \frac{34}{5}$$

The answer is A.

CHECK Your Progress

- 7.** If $5y + 2 = \frac{8}{3}$, what is the value of $5y - 6$?

F $-\frac{20}{3}$

G $-\frac{16}{3}$

H $\frac{16}{3}$

J $\frac{32}{3}$



Personal Tutor at algebra2.com

To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.



Real-World Link

Previously occupied homes account for approximately 85% of all U.S. home sales. Most homeowners remodel within 18 months of purchase. The top two remodeling projects are kitchens and baths.

Source: National Association of Remodeling Industry

Real-World EXAMPLE Write an Equation

8 HOME IMPROVEMENT Josh spent \$425 of his \$1685 budget for home improvements. He would like to replace six interior doors next. What can he afford to spend on each door?

Explore Let c represent the cost to replace each door.

Plan Write and solve an equation to find the value of c .

The number of doors	times	the cost to replace each door	plus	previous expenses	equals	the total cost.
6	•	c	+	425	=	1685

Solve $6c + 425 = 1685$ Original equation

$6c + 425 - 425 = 1685 - 425$ Subtract 425 from each side.

$6c = 1260$ Simplify.

$\frac{6c}{6} = \frac{1260}{6}$ Divide each side by 6.

$c = 210$ Simplify.

Josh can afford to spend \$210 on each door.

Check The total cost to replace six doors at \$210 each is $6(210)$ or \$1260. Add the other expenses of \$425 to that, and the total home improvement bill is $1260 + 425$ or \$1685. Thus, the answer is correct.

CHECK Your Progress

8. A radio station had 300 concert tickets to give to its listeners as prizes. After 1 week, the station had given away 108 tickets. If the radio station wants to give away the same number of tickets each day for the next 8 days, how many tickets must be given away each day?

Math Online Problem Solving Handbook at algebra2.com

CHECK Your Understanding

Example 1
(p. 18)

Write an algebraic expression to represent each verbal expression.

- five increased by four times a number
- twice a number decreased by the cube of the same number

Example 2
(p. 18)

Write a verbal expression to represent each equation.

- $9n - 3 = 6$
- $5 + 3x^2 = 2x$

Example 3
(p. 19)

Name the property illustrated by each statement.

- $(3x + 2) - 5 = (3x + 2) - 5$
- If $4c = 15$, then $4c + 2 = 15 + 2$.

Examples 4–5
(p. 20)

Solve each equation. Check your solution.

7. $y + 14 = -7$

8. $3x = 42$

9. $16 = -4b$

10. $4(q - 1) - 3(q + 2) = 25$

11. $1.8a - 5 = -2.3$

12. $-\frac{3}{4}n + 1 = -11$

Example 6
(p. 21)

Solve each equation or formula for the specified variable.

13. $4y - 2n = 9$, for y

14. $I = prt$, for p

Example 7
(p. 21)

15. **STANDARDIZED TEST PRACTICE** If $4x + 7 = 18$, what is the value of $12x + 21$?

A 2.75

B 32

C 33

D 54

Example 8
(p. 22)

16. **BASEBALL** During the 2005 season, Jacque Jones and Matthew LeCroy of the Minnesota Twins hit a combined total of 40 home runs. Jones hit 6 more home runs than LeCroy. How many home runs did each player hit? Define a variable, write an equation, and solve the problem.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
17–22	1
23–26	2
27–30	3
31, 32	4
33–36	5
37–40	6
41	7
42, 43	8

Write an algebraic expression to represent each verbal expression.

17. the sum of 5 and three times a number

18. seven more than the product of a number and 10

19. four less than the square of a number

20. the product of the cube of a number and -6

21. five times the sum of 9 and a number

22. twice the sum of a number and 8

Write a verbal expression to represent each equation.

23. $x - 5 = 12$

24. $2n + 3 = -1$

25. $y^2 = 4y$

26. $3a^3 = a + 4$

Name the property illustrated by each statement.

27. If $[3(-2)]z = 24$, then $-6z = 24$.

28. If $5 + b = 13$, then $b = 8$.

29. If $2x = 3d$ and $3d = -4$, then $2x = -4$.

30. If $y - 2 = -8$, then $3(y - 2) = 3(-8)$.

Solve each equation. Check your solution.

31. $2p = 14$

32. $-14 + n = -6$

33. $7a - 3a + 2a - a = 16$

34. $x + 9x - 6x + 4x = 20$

35. $27 = -9(y + 5) + 6(y + 8)$

36. $-7(p + 7) + 3(p - 4) = -17$

Solve each equation or formula for the specified variable.

37. $d = rt$, for r

38. $x = \frac{-b}{2a}$, for a

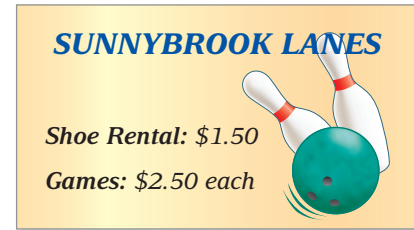
39. $V = \frac{1}{3}\pi r^2 h$, for h

40. $A = \frac{1}{2}h(a + b)$, for b

41. If $3a + 1 = \frac{13}{3}$, what is the value of $3a - 3$?

For Exercises 42 and 43, define a variable, write an equation, and solve the problem.

- 42. BOWLING** Omar and Morgan arrive at Sunnybrook Lanes with \$16.75. What is the total number of games they can afford if they each rent shoes?
- 43. GEOMETRY** The perimeter of a regular octagon is 124 inches. Find the length of each side.

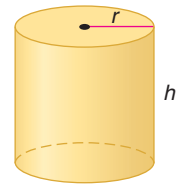


Write an algebraic expression to represent each verbal expression.

- 44.** the square of the quotient of a number and 4
- 45.** the cube of the difference of a number and 7

GEOMETRY For Exercises 46 and 47, use the following information.

The formula for the surface area of a cylinder with radius r and height h is π times twice the product of the radius and height plus twice the product of π and the square of the radius.



- 46.** Write this as an algebraic expression.
- 47.** Write an equivalent expression using the Distributive Property.

Write a verbal expression to represent each equation.

- 48.** $\frac{b}{4} = 2(b + 1)$
- 49.** $7 - \frac{1}{2}x = \frac{3}{x^2}$

Solve each equation or formula for the specified variable.

- 50.** $\frac{a(b-2)}{c-3} = x$, for b
- 51.** $x = \frac{y}{y+4}$, for y

Solve each equation. Check your solution.

- 52.** $\frac{1}{9} - \frac{2}{3}b = \frac{1}{18}$
- 53.** $3f - 2 = 4f + 5$
- 54.** $4(k + 3) + 2 = 4.5(k + 1)$
- 55.** $4.3n + 1 = 7 - 1.7n$
- 56.** $\frac{3}{11}a - 1 = \frac{7}{11}a + 9$
- 57.** $\frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{7}x$

Cross-Curricular Project

Math Online You can write and solve equations to determine the monthly payment for a home. Visit algebra2.com to continue work on your project.

For Exercises 58–63, define a variable, write an equation, and solve the problem.

- 58. CAR EXPENSES** Benito spent \$1837 to operate his car last year. Some of these expenses are listed at the right. Benito's only other expense was for gasoline. If he drove 7600 miles, what was the average cost of the gasoline per mile?

Operating Expenses	
Insurance:	\$972
Registration:	\$114
Maintenance:	\$105



- 59. SCHOOL** A school conference room can seat a maximum of 83 people. The principal and two counselors need to meet with the school's student athletes to discuss eligibility requirements. If each student must bring a parent with them, how many students can attend each meeting?



Real-World Career

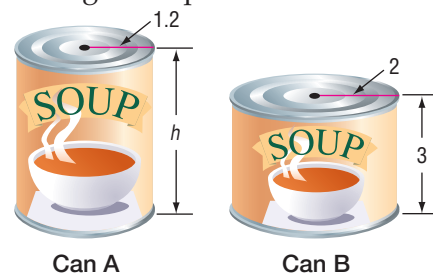
Industrial Design

Industrial designers use research on product use, marketing, materials, and production methods to create functional and appealing packaging designs.



For more information, go to algebra2.com.

- 60. AGES** Chun-Wei's mother is 8 more than twice his age. His father is three years older than his mother is. If the three family members have lived a total of 94 years, how old is each family member?
- 61. SCHOOL TRIP** A Parent Teacher Organization has raised \$1800 to help pay for a trip to an amusement park. They ask that there be one adult for every five students attending. Adult tickets are \$45 and student tickets are \$30. If the group wants to take 50 students, how much will each student need to pay so that adults agreeing to chaperone pay nothing?
- 62. BUSINESS** A trucking company is hired to deliver 125 lamps for \$12 each. The company agrees to pay \$45 for each lamp that is broken during transport. If the trucking company needs to receive a minimum payment of \$1364 for the shipment to cover their expenses, find the maximum number of lamps they can afford to break during the trip.
- 63. PACKAGING** Two designs for a soup can are shown at the right. If each can holds the same amount of soup, what is the height of can A?



RAILROADS For Exercises 64–66, use the following information.

The First Transcontinental Railroad was built by two companies. The Central Pacific began building eastward from Sacramento, California, while the Union Pacific built westward from Omaha, Nebraska. The two lines met at Promontory, Utah, in 1869, approximately 6 years after construction began.

- 64.** The Central Pacific Company laid an average of 9.6 miles of track per month. Together the two companies laid a total of 1775 miles of track. Determine the average number of miles of track laid per month by the Union Pacific Company.
- 65.** About how many miles of track did each company lay?
- 66.** Why do you think the Union Pacific was able to lay track so much more quickly than the Central Pacific?
- 67. MONEY** Allison is saving money to buy a video game system. In the first week, her savings were \$8 less than $\frac{2}{5}$ the price of the system. In the second week, she saved 50 cents more than $\frac{1}{2}$ the price of the system. She was still \$37 short. Find the price of the system.

EXTRA PRACTICE
See pages 891, 926.
Math online
Self-Check Quiz at algebra2.com

H.O.T. Problems

- 68. FIND THE ERROR** Crystal and Jamal are solving $C = \frac{5}{9}(F - 32)$ for F . Who is correct? Explain your reasoning.

Crystal

$$C = \frac{5}{9}(F - 32)$$

$$C + 32 = \frac{5}{9}F$$

$$\frac{9}{5}(C + 32) = F$$

Jamal

$$C = \frac{5}{9}(F - 32)$$

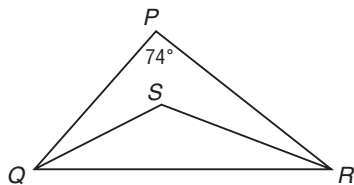
$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F$$

69. **OPEN ENDED** Write a two-step equation with a solution of -7 .
70. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
Dividing each side of an equation by the same expression produces an equivalent equation.
71. **CHALLENGE** Compare and contrast the Symmetric Property of Equality and the Commutative Property of Addition.
72. **Writing in Math** Use the information about ERA on page 18 to find the number of earned runs allowed for a pitcher who has an ERA of 2.00 and who has pitched 180 innings. Explain when it would be desirable to solve a formula like the one given for a specified variable.

STANDARDIZED TEST PRACTICE

73. **ACT/SAT** In triangle PQR , $\overline{QS}$ and $\overline{SR}$ are angle bisectors and angle $P = 74^\circ$. How many degrees are there in angle QSR ?



- A 106 C 125
B 121 D 127

74. **REVIEW** Which of the following best describes the graph of the equations below?

$$8y = 2x + 13$$

$$24y = 6x + 13$$

- F The lines have the same y -intercept.
G The lines have the same x -intercept.
H The lines are perpendicular.
J The lines are parallel.

Spiral Review

Simplify each expression. (Lesson 1-2)

75. $2x + 9y + 4z - y - 8x$

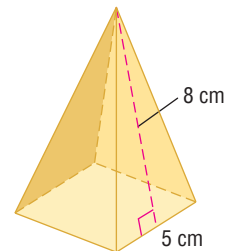
76. $4(2a + 5b) - 3(4b - a)$

Evaluate each expression if $a = 3$, $b = -2$, and $c = 1.2$. (Lesson 1-1)

77. $a - [b(a - c)]$

78. $c^2 - ab$

79. **GEOMETRY** The formula for the surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base. Find the surface area of the square pyramid at the right. (Lesson 1-1)



GET READY for the Next Lesson

PREREQUISITE SKILL Identify the additive inverse for each number or expression. (Lesson 1-2)

80. 2.5

81. $\frac{1}{4}$

82. $-3x$

83. $5 - 6y$